Ergodic Rate Analysis of Massive MIMO Systems in K-Fading Environment

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***Abstract*—Massive MIMO (multiple-input multiple-output) has been identified as a key technology for next generation cellular systems. This paper considers a multi-cellular system with large antenna arrays at the base station (BS) and single antenna user terminals (UTs), operating in a time division duplex (TDD) mode, under a composite fading-shadowing environment. In the uplink transmission, the pilot contamination occurs as the UTs transmit pilots to their respective BSs, and the serving BS estimates the channel state information using a minimum mean squared error estimation. This channel information is further used to design beamforming (BF) and regularized zero-forcing (RZF) precoders for downlink (DL) transmission. We analyze the ergodic rates for DL transmission using different precoding schemes and varying shadowing intensity. It has been observed that shadowing does not average out as we increase the number of antennas as opposed to multi-path fading, and the severity of shadowing badly affects the performance of massive MIMO systems.**

***Index Terms*—Massive MIMO, beamforming, composite fading, K-distribution, MMSE estimation, regularized zero forcing**

I. Introduction

The ever increasing demand of high data rates has initiated a spark in both academia and industry to work on the next generation of cellular systems, namely the fifth generation (5G) networks. 5G will be envisioned with a multitude of technologies such as device-to-device, millimeter wave communications, multi radio-access technologies (RATs) and heterogeneous topologies, and the very large-scale multiple-input multiple-output (MIMO) systems, also known as *Massive MIMO*. A massive MIMO system uses hundreds of antennas at the base station (BS) to simultaneously serve tens of users. It provides high throughput [1], [2], allowing to reap all the benefits of a conventional MIMO on a larger scale by using cheap antennas, thereby increasing data rates and improving the energy efficiency [4]. The large antenna arrays reduce the uplink (UL) and downlink (DL) transmit power through coherent combining, thus achieving higher data rates. In a massive MIMO system, the user terminals (UTs) do not require any channel state information (CSI), however the BS acquires CSI and uses precoding schemes to nullify the effects of channel, thus reducing the overhead on UTs. For DL precoding, CSI is acquired through UL training [10], where the length of training sequence is dependent on the number of users and not on the number of antennas, thus additional antennas do not cause any overhead.

In [1], [5] and [10], the rate analysis in the presence of pilot contamination and inter-cell interference over small-scale fading channel is performed, ignoring the effects of large-scale fading. However, for massive MIMO systems, it has been observed that the small-scale fading is averaged out due to the presence of a large number of antennas, however, the large-scale fading, known as shadowing, still remains a challenge for realizing a practical massive MIMO system. The combined effect of large and small-scale fading is generally modeled by Rayleigh-lognormal (RL) product distribution [16]. However, the rate analysis over composite fading-shadowing channel requires the probability density function (PDF) of the signal-to-interference plus noise ratio (SINR). This PDF is not available in closed-form and hence approximation techniques are required, e.g., [6]-[8]. To study the effects of shadowing, [3] used the generalized-K fading channels for capacity analysis while [6] and [9] used the composite RL model for outage analysis. However, all of these works have analyzed the uplink transmissions only where [6] has assumed perfect CSI, while [3] has taken into consideration both cases, i.e., perfect and imperfect CSI. To the best of knowledge of authors, downlink scenario with BS precoding design has not been reported in the literature over a composite channel.

In this paper, we consider a composite fading-shadowing channel, modeled by K-distribution, which is a combination of Rayleigh and gamma distributions [13], with imperfect CSI estimated by uplink training symbols. This work provides a simple model for composite fading-shadowing environment, unlike other models in which two RVs are multiplied to represent both fading and shadowing, which mostly ends in complicated mathematical forms. In [3] and [6], the authors restrict attention only to UL transmission. In this work, we consider both UL and DL aspects. We analyze the rates for DL transmission by assuming channel reciprocity, thus imperfect CSI will be utilized in designing DL precoders. In the UL, we estimate the CSI using minimum mean squared error (MMSE), whereas after obtaining the estimates of CSI, the BS precodes the downlink data using beam-forming (BF) and regularized zero forcing (RZF) precoders. We present DL ergodic rate analysis for both precoders by using simulations and show that severe shadowing in the environment limits the performance of the massive MIMO system under consideration.

The rest of the paper is organized as follows. Section II describes the massive MIMO system model, in which we define the UL and DL data transmission methods. Section III describes channel estimation and downlink data rates using different linear precoders. Section IV provides the numerical results and discussions for different scenarios. Finally, this paper concludes the work and provides future suggestions in section V.

II. System Model

Consider a multi-cellular system consisting of L cells, where each cell contains a single base station (BS) equipped with M antennas. There are K single antenna user terminals (UTs) distributed uniformly in each cell. This work consider transmission over a composite fading-shadowing environment, which implies that the wireless medium is characterized by both small fading and large-scale fading. Since all the users share same frequency resources simultaneously, the phenomenon of *pilot contamination* occurs at the serving BS, where the desired user's pilots are interfered by the pilots of neighbouring cells' users [5]. This paper consider the uplink training of the BS in a time division duplex (TDD) mode, where channel reciprocity can be used in the downlink. All channel coefficients are assumed to be independent and identically distributed (i.i.d) random variables (RVs), the envelope of whom follows a K-distribution. We consider the following uplink and downlink transmission models.

A. Uplink Transmission

The received signal vector $\mathbf{y}_j^{ul} \epsilon \mathbb{C}^{M\times 1}$ at base station j is given by

$$\mathbf{y}_j^{ul} = \sqrt{\rho_{ul}}\sum_{l=1}^{L}\mathbf{H}_{jl}\mathbf{x}_l^{ul} + \mathbf{n}_j^{ul}, \tag{1}$$

where $\mathbf{H}_{jl} = [\mathbf{h}_{jl1},\ldots,\mathbf{h}_{jlK}] \epsilon \mathbb{C}^{M\times K}$ is the channel matrix of K users from lth cell to jth cell, for any kth user and $\mathbf{h}_{jlk} \epsilon \mathbb{C}^{M\times 1}$ is the channel vector in cell l to base station j. The transmitted symbol matrix of K users and noise vector in the jth cell are $\mathbf{x}_l^{ul} = [x_{l1}^{ul},\ldots,x_{lK}^{ul}]^T$ and $\mathbf{n}_j^{ul} \sim CN(0,\mathbf{I}_M)$, respectively and $\rho_{ul} > 0$ is the uplink transmit power. Each element of $\mathbf{h}_{jlk}$ is a product of i.i.d zero-mean complex Gaussian RV and square root of a gamma distributed RV given by

$$h = \sqrt{z}(g_R + jg_I) \quad ; \quad h \,\epsilon\, \mathbf{h}_{jlk} \tag{2}$$

where g_R, g_I are real and imaginary parts of a complex Gaussian RV and z denotes a gamma RV. The envelope of the channel coefficients eventually follows the so-called K-distribution whose probability density function (PDF) is given by

$$f_{|h|}(|h|) = \frac{2}{\alpha\Gamma(\nu+1)}\left(\frac{|h|}{2\alpha}\right)^{\nu+1} K_\nu\left(\frac{|h|}{\alpha}\right), \tag{3}$$

where α indicates the scale parameter and ν denotes shape parameter. The shape parameter, ν, defines the severity of shadowing in our case. The intensity of shadowing increases for small values of ν and as $\nu\to\infty$, the effects of shadowing decreases and the distribution converge to Rayleigh fading distribution. In (3), $\Gamma(.)$ denotes the gamma function and $K_\nu(.)$ represents the modified Bessel function of second kind with order ν.

For the sake of simplicity, the envelope of the channel coefficients for the serving cell, i.e, $l = 1$ are assumed to follow K-distribution with shape parameter ν_o, whereas for $l = \{2,\ldots,L\}$, the fading envelope follows K-distribution with identical shape parameter ν_I. Hence, we assume that the shadowing severity remains the same from all co-channel cells.

B. Downlink Transmission

The received signal y_{jm}^{dl} at the mth user terminal in the jth cell is given as

$$y_{jm}^{dl} = \sqrt{\rho_{dl}}\sum_{l=1}^{L}\mathbf{h}_{ljm}^H\mathbf{s}_l + n_{jm}^{dl}, \tag{4}$$

where $\mathbf{s}_l \epsilon \mathbb{C}^{M\times 1}$ is the transmit vector of BS l, $n_{jm}^{dl} \sim CN(0,1)$ is additive white Gaussian noise (AWGN) and $\rho_{dl} > 0$ is transmit power of the jth BS. As mentioned earlier, we assume channel reciprocity, i.e., the downlink channel is the Hermitian transpose of uplink channel and the transmitted symbol vector is given by

$$\mathbf{s}_l = \sqrt{\varphi_l}\mathbf{W}_l\mathbf{x}_l^{dl}, \tag{5}$$

where $\mathbf{W}_l = [\mathbf{w}_{l1}\ldots,\mathbf{w}_{lk}] \epsilon \mathbb{C}^{N\times K}$ is a precoding matrix applied over the data symbols $\mathbf{x}_l = [x_{l1}^{dl},\ldots,x_{lk}^{dl}] \epsilon \mathbb{C}^K$ for K UTs in cell l. The φ_l normalizes the average transmit power per UT of BS l, given by

$$\varphi_l = \frac{1}{\mathbb{E}\left[\frac{1}{k}tr(\mathbf{W}_l\mathbf{W}_l^H)\right]}, \tag{6}$$

where $\mathbb{E}(.)$ denotes the expectation operator and $tr(.)$ defines the trace of matrix.

III. Channel Estimation and Downlink Precoding

Each BS estimates $\hat{\mathbf{H}}_{jj}$ of its local channel $\mathbf{H}_{jj}$ by receiving orthogonal pilots from the UTs. Since other BSs are also estimating their local channels through an identical process by using same orthogonal pilots, the serving BS gets pilot contamination, which corrupts the estimation process. The jth BS estimates the channel vector $\mathbf{h}_{jjk}$ for the kth user in jth cell, and the received pilot $\mathbf{y}_{jk}$ is given by[1]

$$\mathbf{y}_{jk} = \mathbf{h}_{jjk} + \sum_{l\neq j}\mathbf{h}_{jlk} + \frac{1}{\sqrt{\rho_{tr}}}\mathbf{n}_{jk}, \tag{7}$$

where $\mathbf{h}_{jlk}$ is interfering channel of kth user from cell l to cell j, and ρ_{tr} is the transmit power of UT k.

[1]We assumed all-one vector for transmission, therefore, we omit the signal $\mathbf{x}_l$ from here onwards for ease of presentation.

To estimate $\hat{\mathbf{h}}_{jjk}$ of $\mathbf{h}_{jjk}$, the classical minimum mean square error (MMSE) estimator is given as

$$\hat{\mathbf{h}}_{jjk} = \mathbf{R}_{jjk}\mathbf{Q}_{jk}^{-1}\mathbf{y}_{jk}^{tr}, \tag{8}$$

where $\mathbf{Q}_{jk} \ \epsilon \ \mathbb{C}^{M\times M}$ and $\mathbf{R}_{jjk} \ \epsilon \ \mathbb{C}^{M\times M}$ are the auto-correlation and cross correlation matrices, respectively. The autocorrelation matrix $\mathbf{Q}_{jk}$ is given by

$$\begin{aligned}\mathbf{Q}_{jk} &= \mathbb{E}\left[\mathbf{y}_{jk}\mathbf{y}_{jk}^{H}\right] \\ &= \mathbb{E}\left[\mathbf{h}_{jjk}\mathbf{h}_{jjk}^{H}\right] + \sum_{l\neq j}^{L}\mathbb{E}\left[\mathbf{h}_{jlk}\mathbf{h}_{jlk}^{H}\right] + \frac{1}{\sqrt{\rho_{tr}}}\mathbf{I}_{M}. \end{aligned}\tag{9}$$

From (9), we have $\mathbf{Q}_{jk} = \zeta\mathbf{I}_M$, where $\mathbf{I}_M$ is $M\times M$ identity matrix and ζ is given as

$$\zeta = \left((\nu_o+1)(2\alpha)^2 + (L-1)(\nu_I+1)(2\alpha)^2 + \frac{1}{\rho_{tr}}\right). \tag{10}$$

Similarly, the cross-correlation matrix $\mathbf{R}_{jjk}$ is given as

$$\mathbf{R}_{jjk} = \mathbb{E}\left[\mathbf{h}_{jjk}\mathbf{y}_{jk}^{H}\right]. \tag{11}$$

From (11), we have

$$\mathbf{R}_{jjk} = (\nu_o+1)(2\alpha)^2\mathbf{I}_M. \tag{12}$$

It can be noticed from (8) that the channel decoupling property can be used such that $\mathbf{h}_{jjk} = \hat{\mathbf{h}}_{jjk} + \tilde{\mathbf{h}}_{jjk}$, where $\tilde{\mathbf{h}}_{jjk}$ is the channel estimation error distributed as a complex normal RV with zero mean and covariance $\mathbf{Z}_{jk}$. Using (9) and (11), we have the following co-variance matrix of the channel estimation error

$$\begin{aligned}\mathbf{Z}_{jk}^{ul} &= \mathbb{E}\left[\tilde{\mathbf{h}}_{jjk}\tilde{\mathbf{h}}_{jjk}^{H}\right], \\ &= \mathbf{R}_{jjk} - \boldsymbol{\phi}_{jjk}, \end{aligned}\tag{13}$$

where we define

$$\boldsymbol{\phi}_{jjk} = \mathbf{R}_{jjk}\mathbf{Q}_{jk}^{-1}\mathbf{R}_{jjk}. \tag{14}$$

Simplifying (14) and putting the results in (13), we eventually obtain

$$\mathbf{Z}_{jk}^{ul} = \Theta\mathbf{I}_M, \tag{15}$$

where the Θ is given as

$$\Theta = (\nu_o+1)(2\alpha)^2\left(1 - \frac{(\nu_o+1)(2\alpha)^2}{\zeta}\right). \tag{16}$$

A. Downlink rates with linear precoding

Since the channel estimation is computed only at the BS, hence the UTs does not contain knowledge of CSI. Therefore, this paper provide the ergodic achievable rates on the basis of techniques used in [5] and [10]. Here, UTs only have knowledge of the estimated value of the channel, which is effective during the channel coherence time. Hence, extra number of antennas always benefits when using channel reciprocity for DL transmission. This implies that as the number of antennas increases, the accuracy of estimated CSI by UTs improves. For this purpose, we decompose the signal y_{jm}^{dl} received at a UT m from the jth BS given as

$$\begin{aligned}y_{jm}^{dl} &= \sqrt{\rho_{dl}\varphi_j}\mathbb{E}[\mathbf{h}_{jjm}^{H}\mathbf{w}_{jm}]x_{jm}^{dl} \\ &\quad + \sqrt{\rho_{dl}\varphi_j}(\mathbf{h}_{jjm}^{H}\mathbf{w}_{jm} - \mathbb{E}[\mathbf{h}_{jjm}^{H}\mathbf{w}_{jm}])x_{jm}^{dl} \\ &\quad + \sum_{(l,k)\neq(j,m)}\sqrt{\rho_{dl}\varphi_j}\mathbf{h}_{ljm}^{H}\mathbf{w}_{lk}x_{lk}^{dl} + n_{jm}^{dl}, \end{aligned}\tag{17}$$

and assume UT optimistically learned the effective channel. Note that, we have taken downlink channel to be the reciprocal of the uplink channel. The ergodic achievable rate, R_{jm}^{dl}, of UT m in cell j is given as

$$R_{jm}^{dl} = log_2(1+\gamma_{jm}^{dl}), \tag{18}$$

where the SINR γ_{jm}^{dl} is given as

$$\gamma_{jm}^{dl} = \frac{\varphi_j\left|\mathbb{E}[\mathbf{h}_{jjm}^{H}\mathbf{w}_{jm}]\right|^2}{\frac{1}{\rho_{dl}} + \varphi_j var[\mathbf{h}_{jjm}^{H}\mathbf{w}_{jm}] + \sum_{(l,k)\neq(j,m)}\varphi_l\mathbb{E}\left[\left|\mathbf{h}_{ljm}^{H}\mathbf{w}_{lk}\right|^2\right]}, \tag{19}$$

where var stands for the variance parameter. We consider two different linear precoders known as Eigen-Beamforming (BF) $\mathbf{W}_j^{BF}$ and regularized zero-forcing (RZF) $\mathbf{W}_j^{RZF}$, which are defined as

$$\mathbf{W}_j^{BF} = \hat{\mathbf{H}}_{jj}, \tag{20}$$

$$\mathbf{W}_j^{RZF} = \left(\hat{\mathbf{H}}_{jj}\hat{\mathbf{H}}_{jj}^{H} + M\varepsilon_j^{dl}\mathbf{I}_M\right)^{-1}\hat{\mathbf{H}}_{jj}, \tag{21}$$

where $\varepsilon_j^{dl} > 0$ is a regularization parameter. We choose $\varepsilon_j^{dl} = \frac{1}{\rho_{dl}M}$, such that precoder could perfectly nullify the effect of noise present in the system.

IV. Numerical Results

In this section, we demonstrate the effects of changing the shadowing intensity on ergodic achievable rates of massive MIMO system under consideration. Various simulation results under different scenarios such as changing the number of BS antennas, shadowing intensity, and different DL precoding schemes have been obtained, where the uplink channel estimation is done using (8). We assume that ten users are uniformly distributed in a given cell. In all simulation results, the shadowing intensity for interfering cells is assumed unity, i.e., $\nu_I = 1$ implying a severe shadowing environment, the number of cells in a cluster is limited to $L = 7$, $\rho_{tr} = 10$dBm, $\rho_{dl} = 40$dBm and $\alpha = 0.15$, unless otherwise stated.

Fig. 1 shows the ergodic rates of 10 users versus the number of antennas with BF and RZF precoders. We vary the shadowing intensity of serving cell by varying ν_o, where the shadowing intensity decreases as we increase the value of ν_o, which results in increasing the data rates as evident from the figure. It can also be noted that the RZF outperforms BF precoder, as it reduces interference more effectively. We also observe that an increase in the number of antennas does not increase the data rates linearly specially at low ν_o, rather a saturating trend in increase in data rate occurs as we further increase the number of antennas. This shows that shadowing does not average out as we increase the number of antennas in a massive MIMO system. However, it continues to affect the

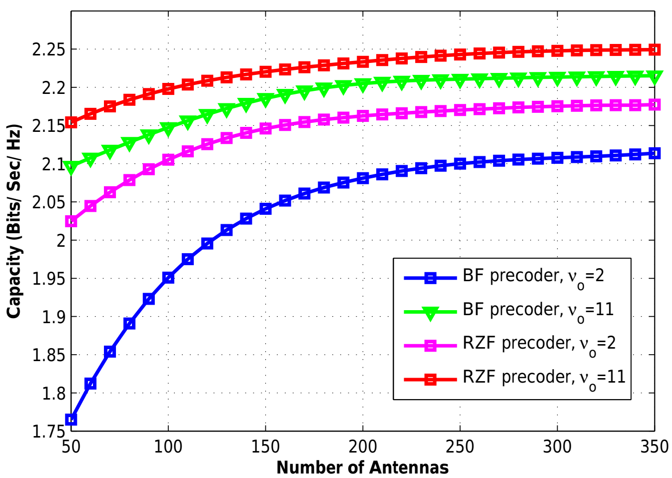

Fig. 1: Downlink rates for different shadowing intensities with RZF, BF precoders, $L = 7$, $K = 10$

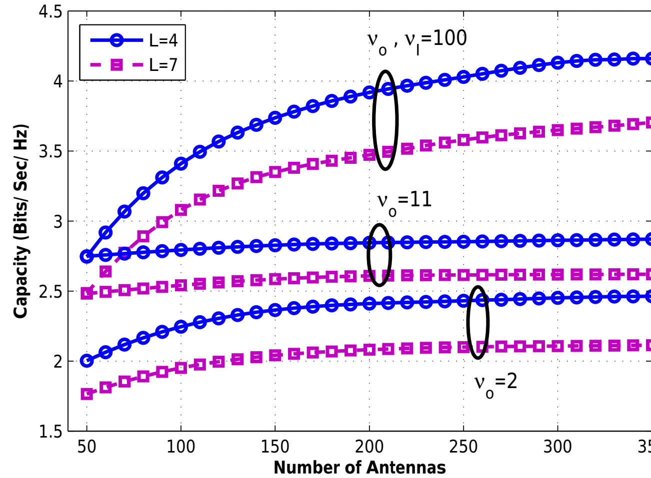

Fig. 3: Downlink rates for different shadowing intensities with BF precoder, $K = 10$

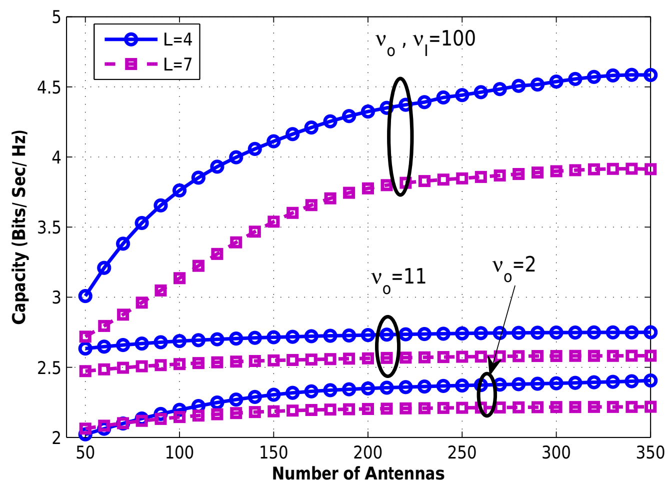

Fig. 2: Downlink rates for different shadowing intensities with RZF precoder, $K = 10$

system even at a larger scale. We can observe from Fig. 1 that the data rate (with BF precoder and $\nu_o = 2$) increases 17% when we increase the number of antennas from 50 to 150, but it only increases by 3.4% as the number of antennas increase from 150 to 350. Thus the rates saturate as we increase the number of antennas in the presence of shadowing, as it limits the performance of massive MIMO system.

Fig. 2 and Fig. 3 depict the ergodic rates of 10 users versus the number of antennas with RZF and BF precoders, and for different values of ν_o and L. It can be shown from the figures that as we increase the cell cluster size, data rates gradually decrease, which occurs due to an increase in the number of interfering cells. Also shadowing effect disappears as $\nu \to \infty$ and the only remaining effect is the Rayleigh fading. Therefore, we can see that at higher values of ν_o and ν_I, i.e., $\nu_o = 100$ and $\nu_I = 100$, which implies both serving cell and the interfering cells are only affected by Rayleigh fading, the performance of system is much better. We compare the rates for this scenario with our conventional scenarios, i.e., with parameter settings of $\nu_o = \{2, 11\}$ and $\nu_I = 1$. It can be shown that with Rayleigh fading only, the massive MIMO system performs better. The data rates dramatically increase as the number of antenna increases in the Rayleigh fading only scenario.

Fig. 4 shows the trend in the estimation error variance (Θ) for different cluster sizes, i.e, $L = \{4, 7\}$ as a function of ν_o. It can be observed that the theoretical values of Θ obtained via (16) are in agreement with the numerical simulations. Note that K-distribution becomes Rayleigh distribution as $\nu \to \infty$, hence as we increase ν_o, the value of channel estimation variance also saturates. The saturation is observed more prominently for $L = 4$ rather than $L = 7$ due to less inter-cell interference.

Fig. 5 shows the ergodic rates for 10 users as a function of the number of antennas and ν_o, with RZF precoder. The result quantifies the number of antennas required to achieve a specific data rate at a certain ν_o. For instance, to obtain data rate of 2.21, a massive MIMO requires 150 antennas for a moderately shadowed environment with $\nu_o = 9$. However, to maintain the same rate, the same system requires 250 BS antennas for a highly shadowed system with $\nu_o = 2$. Therefore, consideration of large-scale fading in designing of future massive MIMO systems is critical.

V. Conclusion

This paper analyzed the effect of shadowing on the ergodic rates of a multi-cell, multi-user massive MIMO system. The uplink CSI for a K-fading environment has been estimated using MMSE estimation, whereas the ergodic rates of DL transmission for RZF and BF precoding schemes have also been provided. The results demonstrated that the shadowing effects cannot be completely averaged out even at a very large number of BS antennas. Motivated from this work, we aim to extend this work to asymptotic rate analysis over a composite

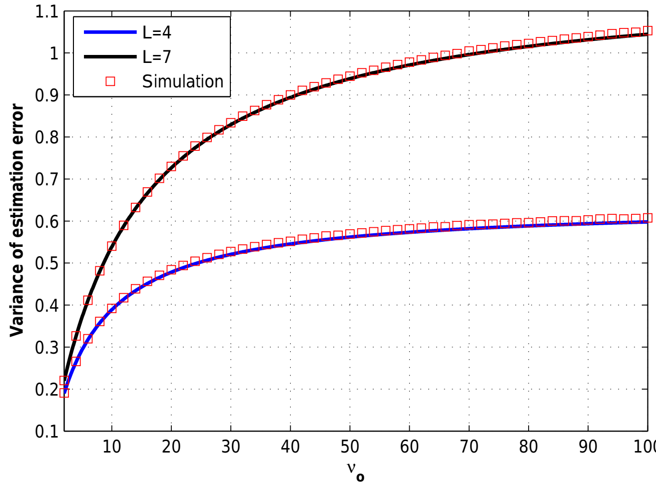

Fig. 4: Estimation error variance for different shape parameters, $K = 10$, $M = 50$.

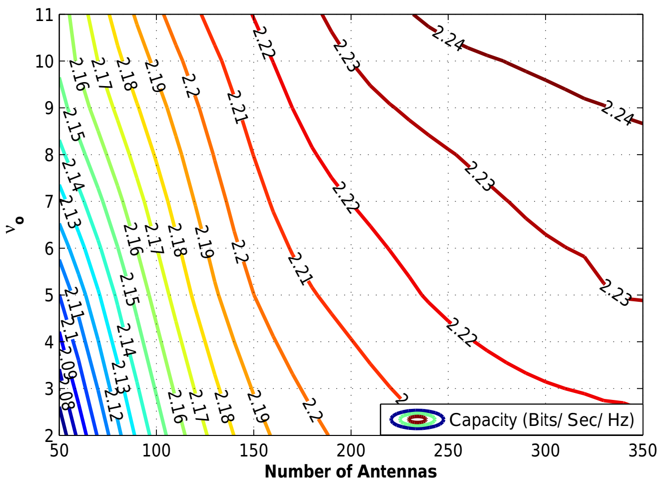

Fig. 5: Downlink rates for RZF precoder as a function of number of antennas and ν_o, $K = 10$

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fading channel under the conditions that number of antennas approach infinity, however a finite ratio between the number of antennas and users in a cell is maintained.

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